ON THE IMPLEMENTATION OF MATHEMATICAL TASKS THAT PROMOTE MATHEMATICAL DISCUSSION: PRE-SERVICE TEACHERS' EXPERIENCES

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Abstract

The aim of this study was to understand what elements related to productive mathematical discussions could be identified by future teachers who participated in the implementation in mathematics courses of learning units for teacher training. Also, we inquired which of these elements they would be willing to replicate in their future teaching practice. The results showed that future teachers identified conditions that promoted mathematical discussion associated with characteristics of the mathematical tasks and principles for leading the discussion. They widely recognized the usefulness of trying to transfer these elements to their future classrooms, and it was even possible to observe cases in which this transfer was actually carried out. These findings have implications for both practice and research.

Introduction

During their initial training, prospective teachers (PTs) must not only develop Mathematical Knowledge for Teaching (MKT), but also fundamental teaching practices (Ball, Thames and Phelps, 2008; Ball & Forzani, 2009). In particular, they are expected to conduct productive mathematical discussions, which has been shown to impact school students' performance (Chapin, O'Connor & Anderson, 2003; Jansen, 2009).

The way in which a mathematics teacher educator (MTE) leads mathematical discussions influences how PTs value and incorporate this practice (Boyd, 2014; Loughran & Berry, 2005). The tasks proposed by the MTE and the types of activities that develop from them, in turn, mediate how classroom discussions take place, providing elements that PTs can project in their future work (Jansen, 2009; Zaslavsky, 2007).

This study is part of a project aimed to improve pre-service primary school teacher education in mathematics. It has two main focuses: i) to increase MKT in PTs through the implementation of Learning Units developed to foster inquiry and the analysis of learning situations; and ii) to support and guide the use of active learning methodologies by MTEs, such as mathematical discussion. The units designed consists of a sequence of lessons that revolve around a high-impact mathematical topic for teacher training.

This project entails research to understand how PTs recognize certain elements regarding the mathematical discussion carried out during the project implementation. Our work addresses the following research questions: a) From the PT's perspective, what aspects that promote mathematical discussion were they able to recognize throughout the implementation of the Learning Units?; and b) Which of these aspects would they be willing to replicate in their future teaching practice?

Framework

Discussion is an essential component in the process of teaching and learning mathematics (Boerst et al., 2011). It is based on the recognition that learning takes place through social interaction (Lave & Wenger, 1991; Vygotsky, 1978) that allows sharing ideas and participating in the collaborative construction of knowledge (Smith & Stein, 2011). Discussions provide teachers with opportunities to elicit students' thinking, which is key when making decisions about teaching (Cirillo, 2013; Pimm, 1987).

Mathematical Discussion is used to describe a modality of classroom interaction, defined by Pirie and Schwarzenberger (2000) as a useful conversation about a mathematical topic, in which there is a genuine contribution and interaction among students. Unlike teaching practices where the interaction proposed by the teacher is focused on assessing the correctness of the students' responses, mathematical discussions focus on students sharing their thoughts with each other. Guiding the discussion is not only intended to make students speak out more, but also to ensure that it is productive with respect to student learning goals (Chapin O'Connor & Anderson, 2003).

A necessary condition to promote productive discussions is the appropriate choice of the mathematical tasks (Smith and Stein, 2011). These tasks must have a high-cognitive demand (Smith and Stein, 1998), yet they must also allow students to participate.

Leading productive discussions requires mathematical knowledge and relevant teaching skills, which entails a great challenge for implementation (Boerst et al., 2011). By breaking down the teaching practice into smaller routines, these authors identified five phases that describe the general structure of a discussion: (1) Designing the task; (2) Monitoring student work; (3) Launching the discussion; (4) Leading the discussion; and (5) Concluding the discussion.

Characteristics of the Learning Units

Several Learning Units were designed throughout the project. Each Learning Unit consists of two 90-minute lessons, with 4 activities sequenced according to the purposes of each class.

The themes of the two units of this study were selected for their high impact for initial training and for the feasibility of being tested by the MTEs. One unit addresses the notion of border and definition of perimeter using previous PT knowledge to develop more sophisticated notions of these concepts (Lu, Weng & Tuo, 2013). The other deals with PT misconceptions

about variations of area and perimeter of geometric shapes (D'Amore & Fandiño, 2007; Ma,1999).

The units are materialized in student worksheets and a description document for the MTE, which includes the lesson plan and purpose, modality, times, most likely answers and mistakes, teaching notes, together with recommendations to carry out the transitions between activities. In addition, suggestions to lead the mathematical discussion were incorporated.

Methodology

This study is built on a pilot of the described Learning Units in disciplinary mathematics courses of two Chilean primary school teacher education programs. It is worth pointing out that MTEs agreed to go through the units according to the requirements of the original plan.

A qualitative research approach was used to understand students' perspectives about their learning experience (Flick, 2002). The participants were 29 volunteer students, distributed as shown in Table 1.

Focus groups were used to collect data, as they facilitate dialogue and discussion among participants, contributing to the exchange of ideas, opinions, and reflections (Kidd & Parshall, 2000). Each focus group was guided by two researchers and included a set of guided questions to understand perceptions about the main learnings achieved, the teaching practices adopted by the MTE, and the elements of the observed practices they would be willing to apply in their own classrooms one day. All the focus groups were recorded and subsequently transcribed.

The preliminary analysis was carried out by two members of the research team, who coded the responses in several emerging categories. These categories were defined through a Constant Comparative Method (Strauss & Corbin, 2007). It should be noted that the first research question was analyzed using the entire data source, while the analysis of the second research question only considered student answers related to the elements of the observed practices they were willing to apply in their classrooms. The members of the research team also cross-checked the results to ensure the reliability of the data.

Results

Following our research questions, we examine below the several categories arising from the data. They were organized in two groups: (1) conditions that promote productive mathematical discussion, and (2) PT's projections of the mathematical discussion to the school classroom.

1) Conditions that promote productive mathematical discussion.

Most of the PTs agreed that the learning experience has contributed to generate conditions that led to productive mathematical discussion. They were able to identify many conditions to

design productive mathematical tasks and for their implementation in the classroom. In many opportunities, this was done by contrasting their experiences in this project with previous mathematical learning experiences, both at school and in university courses.

> *...I think that all or most of us had this upbringing in mathematics while we were studying where it was [just] the teacher, the exercises and you were hardly ever asked for your opinion. In my case, I was afraid of the teacher. So in these classes they kind of gave us the chance to talk and she also, like, made us curious at times about how to keep going (FG Course 1).*

We grouped the conditions perceived by the students into two main categories:

a. Characteristics of the mathematical task

These correspond to attributes of mathematical tasks that foster analysis and reflection among PTs. These characteristics were divided into in three subcategories: encouraging the seek for different solutions, questioning previous tacit knowledge, and sequencing with progressive difficulty. For example:

> *It was very interesting that when someone had a question and asked it out loud, the teacher expected us to answer it ourselves. [So] we would try to explain; and it was very cool because they were very difficult things to explain, because they are, like, very automatic thoughts. Like, I'd never thought about the perimeter like that before (FG Course 1).*

Some PTs mentioned that the tasks allowed them to question their previous conception of the perimeter (see Table 2 for examples of quotes for the other subcategories).

b. Guiding principles for mathematical discussion

The other conditions observed are related to underlying principles for mathematical discussion, such as: not being pressured to give a correct answer, sharing different ideas and positioning, and the need to bring the discussion to a close. Regarding to the first principle, one prospective teacher stated:

> *No pressure of having to [always] give the right answer. Because there's always that, the pressure to answer correctly and if you don't, you're like "OK, I failed". But this wasn't the case in this class, it wasn't necessary. In other words, it was good for us to make mistakes, we needed that to understand (FG Course 4).*

Many students talked about how discussing mathematical ideas required to bridge different, sometimes contrasting, viewpoints. Although they valued this approach, they reported that at times they were not able to reach a shared conclusion and that this left them uncertain about their learning. They relate this difficulty to the ability of MTE to systematize and bring the discussion to a close, as can be seen in the following quote:

> *In the end, I kind of end up doubting what I know too, because [although] sometimes I feel that I'm pretty good at math, I was like "is it like that or not?" and since we never reaching a group consensus of "this is what it is", we were left with doubts about the topic. I ended up having more doubts in class than confidence in what I had learned (FG Course 3).*

2) PTs' projections of mathematical discussion in the school classroom.

All PTs shared a common discourse about the usefulness of replicating certain aspects of the mathematical discussion in their professional practice in schools. Most students talked about projecting things they learnt as something they would like to do in the future. One PT, however, reported to have concretely transferred and applied mathematical discussion during a teaching practice:

> *And that allowed me in practice, when I had to teach, to show the children a picture and ask them why they believed that a fraction is a fraction in which parts are divided unevenly. [...] And I came up with this based on the worksheets that we used (FG Course 4).*

What is most significant about this quote is that the PT was able to design a mathematical task to question students' understandings about fractions, a different topic from those studied in the Learning Units.

Finally, as it can be seen in the quotes in Table 2, PTs were willing to replicate most of the identified conditions to promote mathematical discussions in their teaching work. It is worth noting, however, that no projection regarding the need to bring the discussion to a close was observed.

Discussion

The results of this study show that a sequence of mathematical tasks, accompanied by a detailed planning with well-defined purposes related to mathematical discussion (Yackel, Underwood & Elias, 2007), allow MTEs to enhance PTs' learning outcomes.

The PTs identified characteristic elements of mathematical discussion, even though the MTEs that participated in the experience did not receive specific instructions on it. They acknowledged that, unlike the usual practice of focusing the mathematical work on finding the correct answer, the role of MTEs in whole-class discussions was to make students share their ideas and reason about the possible answers.

They also identified the need for closure as one of the characteristics of the mathematical discussions, recognizing that their purpose is to build learning from the interaction among students (Lave & Wenger, 1991). Nevertheless, they raised concerns, such as the difficulty of dealing with uncertainty during the closing of the discussions, pointing out MTEs as the ones responsible for this. This suggests that it is very important to support MTEs during closing, either through improving the suggestions for discussions or preparing them specifically on how to lead these instances.

Finally, it is striking that the PTs projected the necessary conditions for mathematical discussion into their future teaching work, even though this was not intended in the design of the units. The fact that this arose in the analysis raises a question about how to improve the didactic device to strengthen the bridge between the MTE practices observed throughout the disciplinary mathematical courses and the PTs future teaching work.

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Table 1: Sample of the study.

Course	Teacher Education Program	Participants
1	A	7
2	B	9
3	B	4
4	B	9

Table 2: Summary of thematic coding of the study.

		Examples of conditions that promote productive mathematical discussion.	**Examples of PTs' projections of the mathematical discussion to the school classroom**
Characteristics of the mathematical tasks	**Mathematical tasks that promote the seek of different solutions**	*… so in these classes they kind of gave us an instance to talk and she also kind of made us curious sometimes about how to move forward in the little squares, or seeing your classmates do it in a different way that you hadn't thought of, then oh, there's another way, and here's another. And that's what I liked about these classes, to have this instance and also to learn in a group too, with your classmates (FG Curso 1).*	*I think a lot has to do with participation. To, I don't know, answer the question, go to the board and see how it varies. You can do many exercises in different ways (FG Course 1).*
	Mathematical tasks that question previous tacit knowledge	*What I learned the most when I finally understood, when the teacher made all the combinations and [stated] the difference between area and perimeter, when one increases the other could decrease, that stuck in my head, because until now I had never worked like this, if one increased both increased...I never thought that these things could happen, it became clear to me that things are not always proportional (FG Course 2).*	*And that allowed me in practice, when I had to teach, to show the children a picture and ask them why they believed that a fraction is a fraction in which parts are divided unevenly. And they said, "Yes, yes. It is a fraction" and I would say to them "OK, but under the definition we had earlier, would it still be a fraction?" And I would make them reason and that made their learning more meaningful. [...] And I came up with this based on the worksheets that we used, which allowed us to reason and to bring out our own mental picture of the concept and to modify it, to build something real (FG Course 4).*

	Mathematical tasks sequenced with progressive difficulty	*I thought the timeline was very good, because the exercises started from the very beginning. Maybe at first we did them mostly unaware of what we were doing, but when we moved on to the second or third class, we saw that everything had a purpose, and it was timed like this. I also found that very interesting (FG Course 2).*	*I feel that what I'm keeping, apart from what they said –which I totally agree with– is the thing about concatenation, of the, it's not a great name, but that thing about the "evolution of thought" through a sequence. I have the feeling that there is still something missing, as though I was still waiting for dessert [....] because I want, I want to do this sequencing, but I still lack something for it to take shape so I can grasp it more easily (FG Course 1).*

Table 2: Summary of the sample of the study (continued)

		Examples of conditions that promote productive mathematical discussion.	Examples of PTs' projections of the mathematical discussion to the school classroom
Principles for the implementation of the mathematical discussion	**Not being pressured for correct answers**	*Yes, but for example in the first class, when we were faced with the dilemma of what the correct perimeter was, the teacher never said "this is right and this is wrong", in fact she let the discussion continue and ended the class and said, OK, we will continue next week and see what you decide, but at no time did she say "OK, class dismissed, this is the correct answer", which I think is interesting (FG Course 1).*	*[...] and also, I insist, to be very willing to let different things happen and not to be like "the result, it has to be correct" (FG Course 1).*
	Sharing different ideas, thoughts, and positioning	*I was thinking of analyzing, visualizing, I think that learning to consider the opinion of others [...] how did the teacher do it? Like, he first listens and then asks "how did you do it?", like, he passed the question on to me, I find that's really good. So, even though you may not have the right answer, you are never told "No, this is not right, you are wrong", but instead your colleagues help you shape your learning, I also learned that [...] there were many different views (FG Course 4).*	*I would like to replicate the atmosphere we had during the discussion. I know that in primary education this is a lot more difficult, it's simpler for us adults, but it is something that I would like to replicate, because having different opinions helped us to have a common point between all of us and understand how everyone thought differently, which helps the teacher to understand how to help certain students that find it harder, so to speak. I am more visual, so it was easier for me to see things, but Javiera needs to have something to write about. So having an instance that gave us all of this and then being given time to discuss everything, I find that helped us a lot and it is something*

			I would like to take into my classroom (FG Course 1).
	Need to bring the discussion to a close	*Because later I still worked in a group, and you know that your group has to give an opinion, reach a consensus. At least that which the teacher did to move along, showing the example (of) how the group had arrived to their answer (FG Course 2).* *I could say something similar, but with the difference that she intended to study. I had completely forgotten that I had a test, but when I got to it and saw the questions I was like, "we saw all of this in class, we talked about it" those were multiple choice questions. Then came the open questions, and I saw them and I was like "we talked about this, we discussed it", I could relate everything to what we had experienced, so I felt that we learned more based on the experience talking and reaching a common goal with everyone (FG Course 4).*	**-------**