# A B-LEARNING APPROACH TO DEVELOPING MATHEMATICAL KNOWLEDGE FOR TEACHING FOR IN-SERVICE PRIMARY SCHOOL TEACHERS

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*We describe the development of a blended learning (b-learning) program designed for primary school teachers in Chile. The focus of the program is to provide opportunities to develop teachers' Mathematical Knowledge for Teaching, and it has been elaborated through a collaborative effort between teachers and mathematicians as part of a partnership between a mathematical research center and the Chilean Ministry of Education. We also present some findings regarding course design obtained after a pilot experience with in-service teachers.*

## CONTEXT

In the past 25 years, Chile has experienced rapid economic progress, becoming the first South American country to join the OECD. In The Global Competitiveness Report 2015-2016, by the World Economic Forum (Schwab, 2015), Chile leads Latin American countries and placed 35 in an Index ranking 140 countries. This strongly contrasts with the results shown in education: Chile is placed 108 in quality of primary education and 107 in quality of mathematics and science education. Even more, the report "Chile: Climbing on giants' shoulders: Better school for all Chilean children" by the OCDE (Brandt, 2010) indicates that the country, despite recent improvements as measured by PISA results, still needs to catch up with OECD standards, and to address some important issues in its educational system. As is mentioned in that report, an important factor is to have better-trained teachers.

### The teaching of mathematics in primary school

There is a clear benefit of having a deep expertise in the subject matter in order to teach mathematics. However, many studies have established that mathematical knowledge required for teaching is different from that needed for other professional fields related to mathematics (Ball et al., 2005; Ball et al., 2008; Ma, 1999).

Starting from Shulman's introduction of "pedagogical content knowledge" (Shulman, 1987), and continuing with Liping Ma's (1999) work about the "profound understanding of fundamental mathematics", there has been a great effort to identify and describe the different domains of teachers' knowledge involved in teaching mathematics. Researchers at the University of Michigan, grouped in the project Learning Mathematics for Teaching (Ball et al., 2008) have proposed a model for what they identified as *mathematical knowledge for teaching* (MKT), which is composed of content knowledge and pedagogical content knowledge.

### Primary teachers in Chile

There are important gaps in the disciplinary and pedagogical knowledge in mathematics of pre-service teachers in Chile. The TEDS-M study (Teacher Education and Development Study in Mathematics 2007-2008), applied to 17 countries, placed Chilean teachers in the bottom two in both mathematical knowledge and didactical knowledge, showing more unsatisfactory results than those of less economically developed countries, such as Botswana and the Philippines. This is confirmed by the results of the INICIA, which is a national test that evaluates the content and pedagogical knowledge of pre-service teachers in Chile. These results have raised concern about teachers' training, and have called attention to the need for changes in educational policy around this issue.

Research into the mathematics teaching of in-service teachers in Chile is scarce. However, evidence indicates that teachers grasp the procedural knowledge, but they are not fluent in the conceptual understanding and the specialized content knowledge for teaching mathematics (Rodríguez et al., 2011). This translates into a lack of deductive reasoning during the instruction, and a prioritization of the use of procedures over comprehension of the concepts (Varas et al., 2008; Araya and Dartnell, 2008; Radovic and Preiss, 2010).

Another factor to consider is the teaching load of Chilean teachers, which is intense compared to others countries; this leaves them little time for activities besides teaching, such as lesson planning, collaborative work with peers, and professional development. According to the TALIS 2013 study, Chilean teachers are contracted for an average of 29 hours weekly, of which 27 are spent teaching. This contrasts with the OECD average: 38-hour contracts, with 19 hours spent teaching.

## ICT and b-learning

The most recent ICT census applied in Chile (Mineduc, 2013) has shown that almost all teachers have a computer, and more than 80% have internet access at home. However, school principals have stated that over half of their teachers need ICT training, which is consistent with the low frequency of ICT used in teaching and learning.

Blended learning (b-learning) is a pedagogical approach that combines classroom instruction with the learning opportunities of the online environment. Some studies suggest that b-learning instruction has a significant advantage over purely face-to-face instruction or purely online instruction (Means et al., 2009).

Although there are many advantages of using a b-learning approach for in-service training, there are only a few comprehensive programs in Latin America. One such experience is *Ciencia en tu escuela* (Science at your school), a program in sciences and mathematics promoted by the Mexican Academy of Sciences and developed by the Universidad Nacional Autónoma de México (UNAM).

## DESCRIPTION OF THE PROGRAM

This b-learning program is the product of an alliance between the Chilean Ministry of Education and the Universidad de Chile, and it is led by the Center for Mathematical Modeling, a research center in applied mathematics, which has a large team experienced in mathematics education. The main purpose of the program is to improve teachers' mathematical skills and to deepen their knowledge of fundamental aspects of the teaching of mathematics in primary school. In accordance with the MKT model (Ball et al., 2008), the program is focused on the Common Content Knowledge (CCK) and the Specialized Content Knowledge (SCK). Thus, it promotes: the learning of mathematical concepts, structures and ideas through a phenomenological approach; a conceptual understanding rather than a procedural knowledge; and reflection about the fundamental mathematics for teaching.

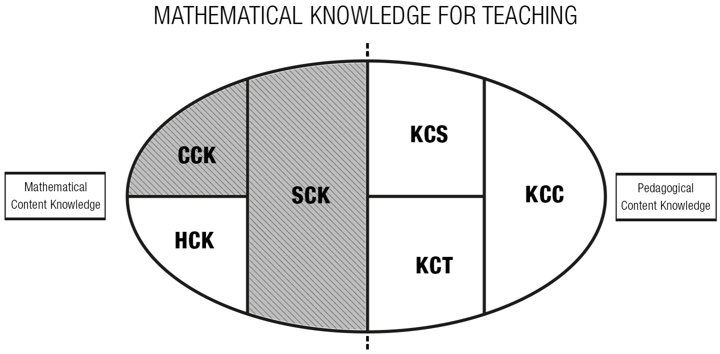


Figure 1: The programme is focused on promoting the development of Common Content Knowledge (CCK) and the Specialized Content Knowledge (SCK) required for teaching mathematics in primary school.

Due to this focus, its development required collaborative work between teachers and mathematicians. To create the scripts and activities, an interdisciplinary team was formed, consisting of teachers with classroom experience and mathematicians with backgrounds in mathematics education. The team also had support from psychologists and instructional designers, who coordinated and managed the learning flow, coherency and user experience, as well as from graphic and web designers who developed the graphics and user interface.

Each script went through a series of quality control instances, where the activities, the writing, and the graphic design were discussed in-depth to evaluate their appropriateness and relevance. After the implementation, each course was tested by a group of elementary teachers, to assess the program's functionality and the activities' clarity.

**Program and course structure**

Although in Chile elementary school teachers are trained as generalists, it is common for teachers specialize and teach only in specific grade levels. Thus, our program is organized according to grade levels and curriculum domains, as is shown in Table 1.

| **Courses** | **Domain** | **Level** |
|---|---|---|
| Numeral systems, addition and subtraction of natural numbers | Numbers | 1º-2º |
| Multiplication and division of natural numbers | Numbers | 3º-4º |
| Representation models, comparison, addition and subtraction of fractions [1] | Numbers | 3º-6º |
| Visualization and properties of two and three-dimensional objects. | Geometry | 3º-6º |
| Multiplication and division of fractions, ratios and percentages [1] | Numbers | 5º-8º |
| Patterns, algebraic language and equations | Algebra | 5º-8º |
| Ratios, percentages, proportions, use and interpretation of graphics [1] | Algebra | 7º-8º |

Table 1: The list of courses of the program ([1] indicates the courses implemented during the pilot). Courses with the following contents are on planning stage: geometric shapes and measurement; area, perimeter and isometric transformations; research cycle and organization, visualization and analysis of data; and uncertainty and probability.

Each course is designed to be taken over 9 weeks, with 4.5 dedicated hours per week. There are 3 face-to-face sessions and 4 to 6 virtual workshops, distributed in two modules (see Figure 2). The online part is asynchronous, that is, teachers work at their own pace; at all times, though, they are monitored and supported by virtual tutors.

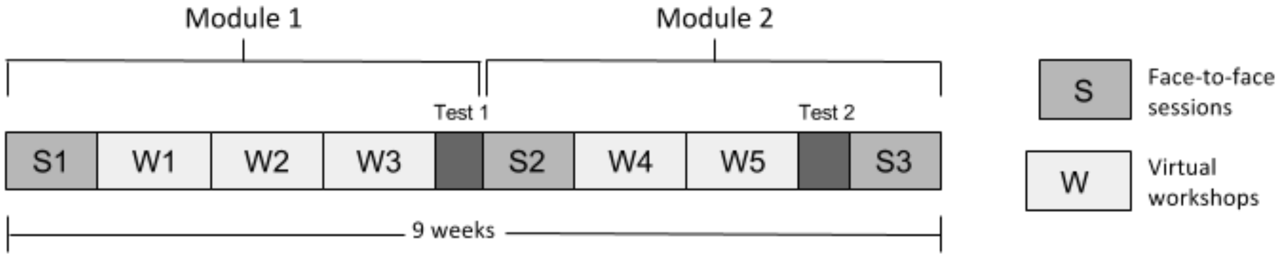


Figure 2: Structure of a course.

The face-to-face sessions are meant to introduce teachers to the program, platform and courses; motivate them to complete the course; answer questions; and systematize and discuss the learning process they are going through.

Most of the teachers' work and learning occurs during the virtual workshops. Each workshop comprises around 4 activities that deal with fundamental aspects of mathematics using a constructivist approach; each activity starts with a contextualized situation in which the mathematical contents appear and are developed step by step. The activities are aimed at promoting: i) mathematical inquiry, through the use of interactive tools; ii) reflection and reasoning, by means of problematic situations, questions and detailed feedback for key questions; iii) the search for different strategies for problem solving; iv) the need for various representations and models, that are illustrated, for example, using diagrams and animations. Many tasks focus on analyzing different strategies and resolution procedures, incomplete solutions, as well as possible errors or difficulties associated with them. In the activities, we privileged the use of short-answer, both numeric and textual, over open questions, and multiple-response over multiple-choice questions. Each activity has instances of synthesis that allow opportunities to organize the mathematical knowledge learned. Additionally, each workshop includes a summary that connects the learning achieved during the activities. We also designed complementary material that further clarifies the mathematical contents addressed over the course and includes some aspects of curricular analysis and assessment.

It is important to remark that although the courses are designed for in-service teacher training, the contexts and most of the activities can be adapted for use in the classroom, while others have the classroom as a context. For this it was crucial to have experienced teachers on the development team.

**Example of a course and tasks**

In this section we briefly describe the structure of the course "Multiplication and division of natural numbers", which is suggested for elementary teachers who teach in 3º and/or 4º grades, and explain the purpose of some of the mathematical tasks used. This course has the following four virtual workshops:

1. **Multiplicative Situations**: the different meanings and interpretations of multiplication and division, definition of these operations and associated representation models.

2. **Multiplication**: properties of multiplication, the construction of the multiplication table, mental strategies and the justification of the standard algorithm to calculate multiplications.
3. **Division**: properties of division, mental strategies and the justification of the standard algorithm to calculate divisions.
4. **Problem Solving**: resolution and analysis of non-routine problems involving multiplication and division, while promoting reflection on some fundamental aspects of problem solving.

The workshops are designed to be developed in about 5 hours and they consist of several activities, each one with a precise learning goal. Through the activities we emphasize the use of representations and the search of general arguments to justify properties, procedures and solutions.

The following example describes the context used in a mathematical task of the first activity of the workshop on Multiplication, whose learning goal is to introduce and make sense of the properties of multiplication.

Example 1: Two siblings are putting together a puzzle. The girl put together the left side of the puzzle and her brother did so with the right side, as it is shown in Fig. 3.

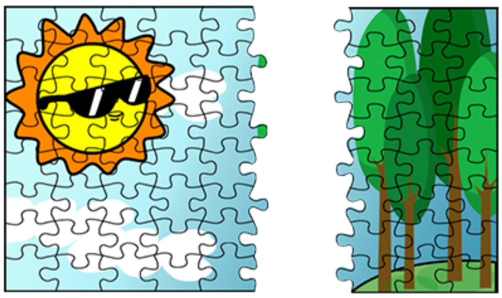

Figure 3: The puzzle in Example 1.

The siblings wonder how to count the total numbers of pieces of the puzzle. The boy notices he can calculate the number of pieces of each part and adding them up to obtain $8 \cdot 5 + 8 \cdot 3$. Her sister, after putting both parts together, concludes that the number of pieces is $8 \cdot (5 + 3)$.

In the task, and through a series of exercises and questions, teachers are assisted in the process of building an argument, using this particular example, to understand why the distributive property of the multiplication makes sense for all natural numbers. This kind of arguments, we refer to as *generic examples,* are used often during the course to show the validity of number properties. Since generic examples allow teachers to illustrate general arguments with concrete examples that involve specific numbers, we promote them as a useful teaching tool to improve students' understanding and sense-making.

The workshop on problem solving has a disctintive structure: each activity revolves around a single problem and serves a dual purpose. On one hand, in the activity teachers are confronted to apply and extend to new situations the contents, representations, strategies and arguments worked in previous workshops. On the other hand, each activity is aimed to promote fundamental aspects of problem solving such as: to incentivate a careful reading of the problem statement to correctly understanding it; to identify the assumptions are required to solve it; to encourage the use of different representations of the problem and resolution strategies; to recognize the error as a learning opportunity; to interpret correctly the results obtained; to acknowledge the importance of answering the problem questions; and to promote the verification of the found solutions. Although teachers have the chance to solve the problem and upload their productions, the aim of each activity is not set in their solutions of the problem, but rather in the analysis and reflection about the different strategies, procedures, interprerpretations and new problems that may appear on its resolution.

In what follows, we present an example of one problem of this worshop and briefly describe some of the tasks used during the activity.

Example 2:

**The tables for the wedding**

Ignacia is organizing a wedding with 54 guests. She has to settle enough tables in order that all guests have a seat. If each table has 8 seats, which is the minimum number of tables that she needs?

Although dividing 54 by 8 is an immediate strategy to solve this problem, the answer is not given by the quotient of this division, but rather this number plus one. After solving this problem, we make teachers to study the same situation with a different number of guests. They can easily observe that in some cases the answer is the quotient of the number of guests divided by 8, but others require adding one to the quotient. Therefore, one "problem solving purpose" of the activity is to emphasize the necessity of interpreting correctly the results of the operations we use to solve a problem.

Later in the activity, teachers are put in a situation where students are drawing diagrams representing different distributions of the guests in a fixed number of tables, as shown in Fig. 4, and they are trying to establish whether the number of tables is the minimum needed or not.

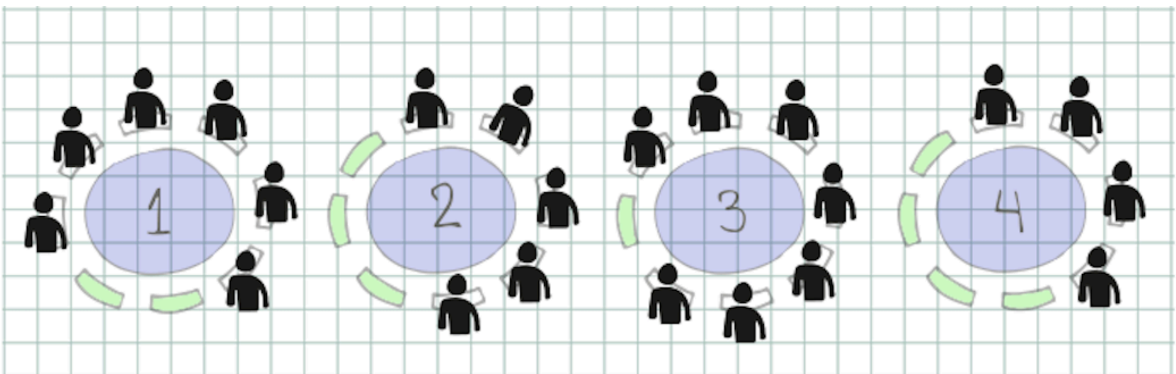


Figure 4: A distribution of 22 guests seated in 4 tables, with 8 seats each one. Can we use less number of tables to seat all the guests? Observe that in this case there are 10 available seats.

The task of the teachers is to identify a sufficient condition to ensure that the number of tables used in a given distribution of guests in the tables is the minimum possible. One such condition is that the number of available seats in the distribution is at most 7, since otherwise there is ways of reorganizing the guests to empty a table. By doing this it is expected that teachers appreciate and reflect on the opportunities that present certain problems to be extended in order to enact students reasoning and improve their understanding.

## RESULTS OF THE PILOT

As part of the development of the program, during 2015 we implemented a pilot of three of the courses with elementary teachers of public schools of four districts in Santiago. The purpose of the pilot was to gather information about the use of the platform and the topics that present greater difficulty for the teachers, and to evaluate the structure of the courses for future improvements.

Several instruments were designed and applied: a questionnarie aimed to obtain an ICT profile of the teachers; a satisfaction survey to evaluate several dimensions; a usability survey to study the perception of the teachers about the platform; and focus groups to gain insight about how teachers worked and felt during the development of the courses, their opinions with respect to relevance of the topics the courses were about and their learning perceptions.

The three courses had a low dropout rate and the overall satisfaction was very high. Teachers reported that the courses were very demanding, appreciated the originality and variety of contexts and manifested that the math contents were developed in a way that is useful for them. Although teachers raised concerns about the difficulty of some of the questions, they acknowledged the diversity of question formats and the focus on reflection. Teachers mentioned that many of the activities could be easily adapted for classroom use; indeed, the fact that classroom lessons based on the courses activities were not provided was a common complaint.

Many interesting findings of this experience were regarding the format of the different components of the virtual workshops, which proved very useful for the courses being developed. For example, the length of the writings came to our attention after several teachers reported having trouble understanding certain questions and explanations, which, according to them, was due to the lack of reading comprehension skill. Therefore, some current improvements have been in the line to shorten the writings, simplify the explanations using more drawings and diagrams, and to present the sections of synthesis in a more user-friendly way.

## CONCLUSIONS AND FURTHER WORK

During the development of the courses, it was crutial the collaboration between math teachers and mathematicians. Each team in charge of the development of a course became involved in many discussions about how to introduce the contents in an intuitive and friendly way, using contexts that are meaningful for teachers and students, while preserving the rigour the teaching of mathematics requires. The review meetings were crucial to define the learning goals of each activity, and they also proved to be very useful for increasing the understanding of elementary mathematics of the team. All this led to the development of better protocols and process for the design of the activities, which resulted in better and richer mathematical tasks.

Considering that one of the main purposes of the program is to satisfy specific teachers' teaching needs, during the pilot emerged the need to design a support system that facilitates teachers to choose appropiately which course to take. Another line the team wants to pursue is the preparation of classroom lessons based on the courses activities and the intective tools. We also plan to design or use existing instruments to measure the impact of the program in the teachers' mathematical knowledge for teaching. Finally, another interesting further development is to adapt the program for pre-service teacher training.

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