Practice Worksheet

Pool testing

Activity 1

A laboratory is developing a new type of test to detect steroid use. In preliminary tests, they have detected that 1 in 20 people who do not consume steroids have a positive result on the test.

1. How can the ratio $\frac{1}{20}$ be interpreted in terms of probabilities?

2. It is stated that the probability that two people who do not use steroids will test positive is $\left(\frac{1}{20}\right)^2$. What assumptions must be met to ensure that this calculation is correct?

3. The laboratory applies the test to a sample of 100 athletes who do not consume steroids and wants to calculate the probability that a certain number of them will give a positive result on the test. What is the random variable being studied?

4. Considering the defined random variable, indicate what it represents and what is the value of each term in the expression $P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}$

5. Explain what the following probabilities represent:

a) $P(X = 2)$

b) $P(X \leq 2)$

c) $P(X \geq 2)$

6. What is the probability that more than one athlete tests positive? Express that probability using binomial coefficients.

Activity 2

Bastián works in the sales area of a telephone company. To offer plans and services, call potential clients by phone. Typically, 30% of the calls he makes end up with the sale of a product or service.

Consider that calls are independent. That is, the result of one call does not affect the result of the following calls.

1. Is it possible to consider the number of sales made in n calls as a binomial experiment? Justify your answer.

2. Consider the random variable X = "number of sales made in 20 calls". What is the probability that k sales have been made?

3. How can we express the probability that Bastian makes at least two sales in 20 calls using binomial coefficients?

Solutions

Act. 1

1. It represents the probability that randomly selecting a person who does not use steroids will give a positive result.

2. Since the multiplicative principle is being used, the assumption was made that both events are independent. That is, it was assumed that the fact that one person consumes steroids does not affect the behavior of the other person.

3. If the random variable is defined as *X*, then we have that *X* = "number of people who test positive."

4. According to the model, *n* is the total number of athletes in the sample, in this case 100; *p* and 1 - *p* represent the probabilities of obtaining a positive and a negative value in the test, in this case $\frac{1}{20}$ y $\frac{19}{20}$, respectively, and *k* represents the value that the random variable takes.

5. a) The probability that exactly two athletes test positive.
b) The probability that at most two athletes test positive.
c) The probability that at least two athletes test positive.

6. The desired probability is $P(X > 1)$, which can be expressed as:

$$1 - \binom{100}{0} \cdot \left(\frac{1}{20}\right)^{0} \cdot \left(\frac{19}{20}\right)^{100} - \binom{100}{1} \cdot \left(\frac{1}{20}\right)^{1} \cdot \left(\frac{19}{20}\right)^{99}.$$

Act. 2

1. Since each call may or may not end in a sale, we can consider each of these as a Bernoulli-type experiment with probability of success *p* = 0,3. Since, furthermore, we have assumed that there is independence between the results of these calls and that the probability of success is constant and equal to *p*, the experiment of making *n* calls and observing the number of successful sales distributes as a binomial with parameters *n* and *p*.

2. The probability is $P(X = k) = \binom{20}{k} \cdot 0,3^{k} \cdot 0,7^{20-k}$.

3. $P(X \leq 2) =$

$$\binom{20}{0} \cdot 0,3^{0} \cdot 0,7^{20} + \binom{20}{1} \cdot 0,3^{1} \cdot 0,7^{19} + \binom{20}{2} \cdot 0,3^{2} \cdot 0,7^{18}$$